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Workshop on Compact Modeling

# An Improved Analytical Solution to the Surface Potential in Un-doped Surrounding-Gate MOSFET



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## *Overview of the talk*

- Surrounding-Gate MOSFET: An Introduction
- Previous Modeling Approaches
- Present Modeling Strategy
- Results Obtained from the Present Analytical Model
- Conclusions



## Surrounding-Gate MOSFET: An Introduction

### Schematic of a Surrounding-Gate MOSFET

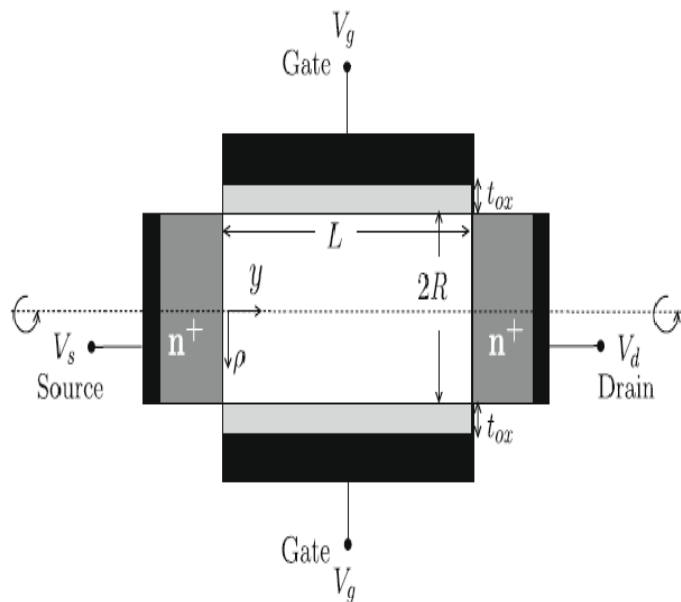


Fig. 1 SRG MOS

[1] J.P.Coling, *Solid State Electron*, 2004, pp:897–905.

### Advantages of Surrounding-Gate structures

- Presence of ultra thin body and ultra thin gate oxide increases transverse electric field, resulting in reduced SCE. This reduction is the maximum in Surrounding-Gate MOS [1]. Ultra-thin body also results in mobility reduction. For Surrounding-Gate MOSFET the need for very thin body is minimum [1].
- An un-doped body results in higher mobility due to less impurity scattering and also reduces random dopant fluctuations.
- Presence of semi-insulating back oxide reduces parasitic capacitances and leakage greatly



## *General Modeling Strategy*

- Solve the Poisson equation in one dimension, under Gradual Channel Approximation.
- Express the potential along radial direction in terms of some implicit state variables and/or the potential at the center.
- Equate the normal electric field at the Si-SiO<sub>2</sub> interface to form an implicit equation of the state variable and/or surface potential.
- Solve the implicit equation (using either numerical or analytical approximation) to evaluate the state variables in terms of device parameters and bias conditions.
- Evaluate surface potential (and/or inversion charge) in terms of the state variable.



## Previous Accomplished Work (I)

$$(V_{GS} - \phi_{ms} - V) - V_T \log\left(\frac{4L_{Di}^2}{R^2}\right) = \frac{Q}{C_{ox}} + V_T \log\left(\frac{Q}{Q_0}\right) + V_T \log\left(\frac{Q+Q_0}{Q_0}\right)$$

$$Q_0 = \frac{4\epsilon_{Si} V_T}{R} \quad V_T = \frac{kT}{q} \quad L_{Di}^2 = \frac{2V_T \epsilon_{Si}}{qn_i} \quad (\text{Debye length})$$

$V \rightarrow$  channel potential,  $R \rightarrow$  cylinder radius,  $\rho \rightarrow$  radial direction,  $\phi_{ms} \rightarrow$  work function difference

$$Q = C_{ox} \left( -\frac{2C_{ox} V_T^2}{Q_0} + \sqrt{\left(\frac{2C_{ox} V_T^2}{Q_0}\right)^2 + 4V_T^2 \log^2\left(1 + \exp\left(\frac{V_{GS} - V_0 - V}{2V_T}\right)\right)} \right) \quad [2] \quad V_0 = \phi_{ms} + V_T \log\left(\frac{4L_{Di}^2}{R^2}\right)$$

*Disadvantage:*

*Deviates significantly in transition region*

[2] B.Iniguez, et al , *IEEE Trans. Electron Devices*, Aug. 2005, pp:1868–1873.



## Previous Accomplished Work (II)

$$C_{ox} \frac{V_{GS} - \phi_{ms} - \phi_s}{V_T} = \frac{R \epsilon_{Si}}{L_{Di}^2} \exp\left(\frac{\phi_0 + \phi_s - 2V}{2V_T}\right)$$

$$\phi_s = V_{GS} - \phi_{ms} - 2V_T W_0 \left( \frac{R \epsilon_{Si}}{2L_{Di}^2 C_{ox}} \exp\left(\frac{V_{GS} + \phi_0 - 2V - \phi_{ms}}{2V_T}\right) \right)$$

[3]  $W_0 \rightarrow$  Lambert Function

$$\phi_0 = \frac{1}{2} \left[ (\phi_{0\max} + \phi_{0a}) - \sqrt{(\phi_{0\max} - \phi_{0a})^2 + \delta \phi_{0\max} \phi_{0a}} \right]$$

$$\phi_{0\max} = V + V_T \ln\left(\frac{4L_{Di}^2}{R^2}\right) \quad \phi_{0a} = V_{GS} - \phi_{ms}$$

*Disadvantage:*

*Use of piecewise model and smoothing functions*

[3] J.He, et al, *Solid State Electron*, July 2007, pp:802–805.



## Previous Accomplished Work (III)

$$\ln \beta - \ln(1 - \beta^2) + s \beta^2 / (1 - \beta^2) - G = 0 \quad \beta \in (0,1)$$

$$s = 2 \varepsilon_{si} \ln(1 + t_{ox} / R) / \varepsilon_{ox} \quad G = (V_{GS} - \phi_{ms} - V) / V_T - \ln(2L_{Di} / R) \quad \text{and } t_{ox} \rightarrow \text{oxide thickness}$$

$$z = \sqrt{\left(\frac{1}{2s^2}\right)^2 + \left(\frac{1}{s^2}\right) \ln^2(1 + e^G)} - \frac{1}{2s^2} \quad [4] \quad z = \frac{\beta^2}{1 - \beta^2}$$

Use correction steps to increase accuracy of the initial guess  $z$  using 2<sup>nd</sup> order Newton Rhapsion method

*Disadvantage:*

*Use of piecewise model and smoothing functions*

*What happens when  $G \approx 0$ ?  $z \approx 0$*

[4] B.Yu, H.Lu, M.Liu and Y.Taur, *IEEE Trans. Electron Device*, Oct. 2007, pp: 2715-2722.



## Previous Accomplished Work (IV)

$$\alpha = \frac{1}{1 + \frac{2}{\eta} W_0 \left( \frac{1}{2} \sqrt{1 - \alpha \eta e^{v/2}} \right)} \quad [5] \quad \alpha = 1 - \beta^2 \quad \beta \text{ defined as in [4]}$$

$$y = z - \frac{z(z-1)}{1+z} + \frac{z(z-1)^2}{2(1+z)^3} - \frac{(z-1)^3(z-2z^2)}{6(1+z)^5} + \frac{z(6z^2-8z+1)(z-1)^4}{24(1+z)^7} - \frac{z(24z^3-58z^2+22z-1)(z-1)^5}{120(1+z)^9} \\ + \frac{z(120z^4-444z^3+328z^2-52z+1)}{720(1+z)^{11}} - \frac{z(720z^5-3708z^4+4400z^3-1452z^2+114z-1)}{5040(1+z)^{13}} \quad z = x/e \\ \text{valid for } x \leq 8$$

$$y = L_1 - L_2 + \frac{L_2}{L_1} + \frac{L_2(L_2-2)}{2L_1^2} + \frac{L_2(6-9L_2+2L_2^2)}{6L_1^3} + \frac{L_2(-12+36L_2-22L_2^2+3L_2^3)}{12L_1^4} + \frac{L_2(60-300L_2+350L_2^2-125L_2^3+12L_2^4)}{60L_1^5}$$

*Disadvantage:*

*Cannot be evaluated analytically*

$L_1 = \ln(x)$   
 $L_2 = \ln \ln(x)$   
 valid for  $x > 8$

[5] H.Morris et al, *IEEE UIGM Proceedings*, June 2006, pp:117-121.



## Present Modeling Strategy

$$\ln \beta - \ln(1 - \beta^2) + s \beta^2 / (1 - \beta^2) - G = 0 \quad 1$$

s is a process parameter and G depends on applied bias as defined in [4] and shown in slide 7

$$\ln \left( \frac{\beta^*}{1 - (\beta^*)^2} \right) + s \left( \frac{\beta^*}{1 - (\beta^*)^2} \right) - G = 0 \quad 2$$

$$y + se^y = G \quad 3$$

$$y = G - W_0(e^{G/s}) \quad 4$$

$$\ln \left( \frac{\beta^*}{1 - (\beta^*)^2} \right) = G - W_0(e^{G/s}) \quad 5$$



## Present Modeling Strategy (contd)

$$\frac{\beta^*}{1 - (\beta^*)^2} = \exp \left[ G - W_0(e^G s) \right] = \frac{1}{k} \quad 6$$

$$W_0(x) = 0.04 + 0.665[1 + 0.0195 \ln(1 + x)] \ln(1 + x) \quad [6] \quad 7$$

$$\beta^* = \frac{2}{k + \sqrt{k^2 + 4}} \quad 8$$

$$f(\beta) = \ln \beta - \ln(1 - \beta^2) + s\beta^2 / (1 - \beta^2) - G \quad 9$$

$$\beta_1 = \beta^* - \frac{f(\beta^*)/f_1(\beta^*)}{1 - f(\beta^*)f_2(\beta^*)/2f_1^2(\beta^*)} \quad 10$$

$f_1 \rightarrow 1^{st}$  derivative and  $f_2 \rightarrow 2^{nd}$  derivative of  $f$  w.r.t.  $\beta$

[6] A.Ringwald et al, QCDINS, *Comput. Phys. Commun.*, Nov. 2000, pp: 267–305.



## Comparison of Correction Methods

Here we show the correction form used in the present model and [7] is more efficient than that in [4]

$$\frac{f(\beta)/f_1(\beta)}{1 - f(\beta)f_1(\beta)/2f_1^2(\beta)}$$

*form of correction method in present model [8]*

$$f(\beta)/f_1(\beta)\left(1 - f(\beta)f_1(\beta)/2f_1^2(\beta)\right)^{-1}$$

$$\approx f(\beta)/f_1(\beta)\left(1 + f(\beta)f_1(\beta)/2f_1^2(\beta)\right)$$

*correction method as in [4], (excluding 3<sup>rd</sup> derivatives)*

$$f(\beta)f_1(\beta)/2f_1^2(\beta) \ll 1$$

*valid only if*

[7] A.R.Boothroyd , *et al*, *IEEE Trans. on CAD of Integrated Circuits and Syst.*, Dec. 1991, pp:1512– 1529.

[8] F.B.Hildebrand, “*Introduction to Numerical Analysis*”, 2<sup>nd</sup> edition, McGraw-Hill.



# Results Obtained from the Analytical Modeling

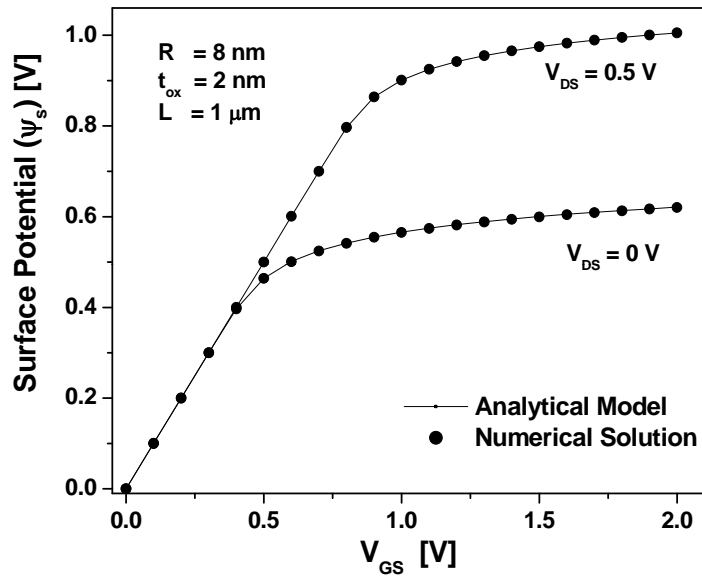


Fig.2. Comparison of surface potential as obtained from the proposed analytical solution and exact numerical method.

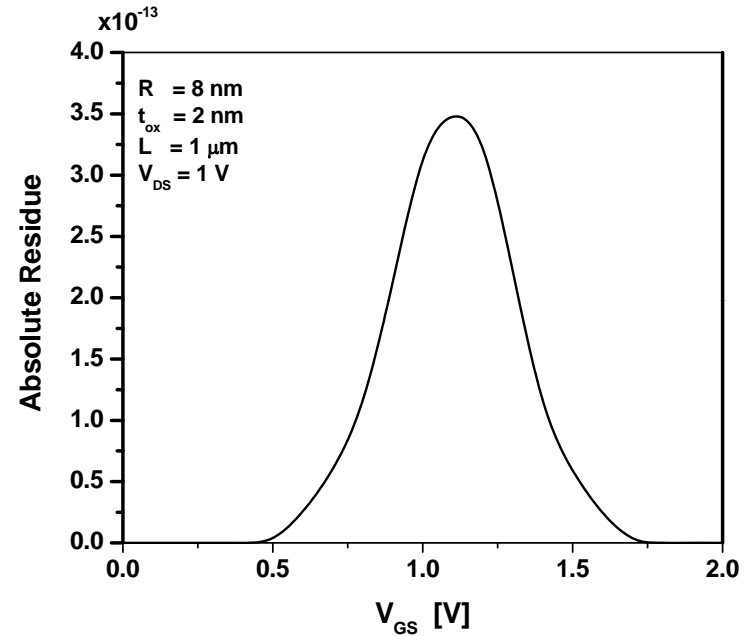


Fig.3. Plot of absolute residue in  $\beta$  values, obtained from plugging the results from our analytical model in (1).

# Results Obtained from the Analytical Modeling (contd)

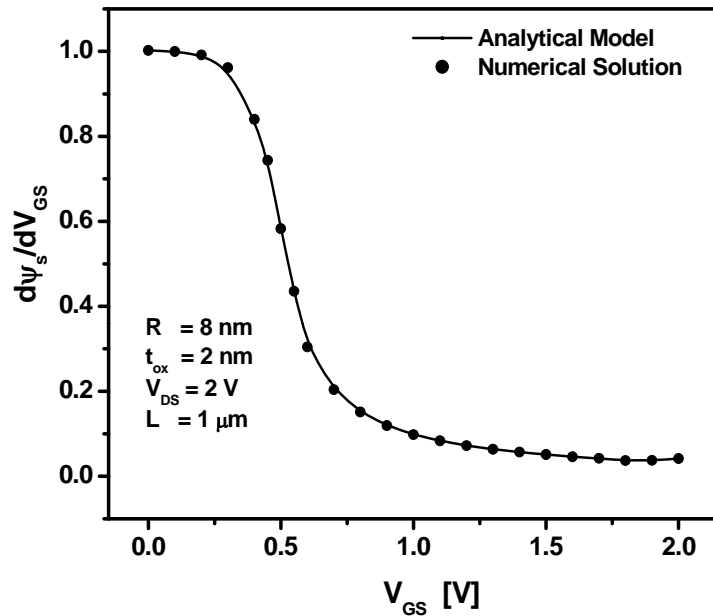


Fig.4. Comparison of first derivative of surface potential obtained from the analytical solution for different values of  $V_{GS}$ , with that from numerical solution.

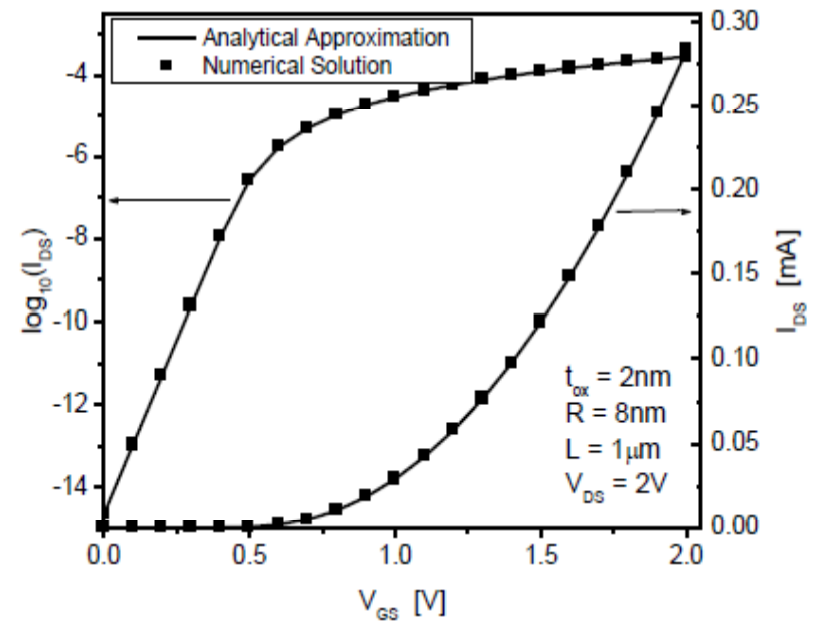


Fig.5. Comparison of  $I_{DS}$  (both log and linear scale) as obtained from surface potential values calculated numerically and from our analytical solution for different  $V_{GS}$  using equations in [9].

[9] G. Dessai, A.Dey, G.Gildenblat and G.D.J.Smit, *Solid State Electron*, May 2009, pp:548-556.



# Results Obtained from the Analytical Modeling (contd)

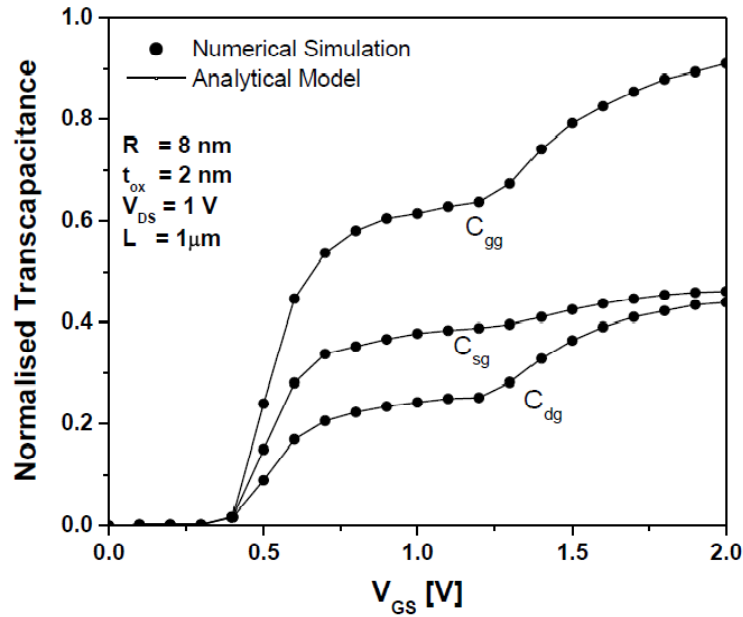


Fig.6. Comparison of normalized transcapacitances obtained from our analytical model and numerical simulations using capacitance equations in [9].

Approximation	CPU time	Fails if $V_{GS}-V_{FB} <$
[4]	0.22	-1.3V
Present Work	0.214	Works in all cases

Table 1. enumerates the computational efficiency and applicability of the present model and that presented in [4].

[4] B.Yu, H.Lu, M.Liu and Y.Taur, *IEEE Trans. Electron Device*, Oct. 2007, pp: 2715-2722.



## Conclusion

A simple analytic solution to the surface potential for an un-doped surrounding-gate MOSFET is presented. The salient features of the present model include

- The solution is numerically robust and is valid in all regions of operation of the device.
- The solution does not use piecewise models and empirical functions to join them.
- The solution is computationally more efficient than that in [4], yet maintaining the same accuracy.

